Short communication

Optimal carrier-smoothed-code algorithm for dual-frequency GPS data

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Abstract

Carrier-smoothed-code (CSC) algorithm is an effective pseudorange multipath mitigation technique, which can alleviate the computational burden and reduce the communication bandwidth needed for the transmission of GPS observations. This paper presents an improved Hatch filter and addresses the optimal CSC algorithm for dual-frequency GPS data based on the optimal parameter estimation theory. The smoothed observations have the same information content as that of the raw dual-frequency GPS data from which these are derived. Consequently, the optimal CSC algorithm is equivalent to the uncombined algorithm. Theoretical analyses show that the data precision of the optimal CSC algorithm is better than that of Hatch filter and its improved version.
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Keywords: GPS; Carrier-smoothed-code algorithm; Hatch filter; Time-invariant combinations

1. Introduction

The carrier phase measurements are extremely precise but biased due to integer ambiguities. Code tracking provides essentially unambiguous pseudoranges which are coarse comparing with the carrier phase. Carrier-smoothed-code (CSC) algorithm, which is an effective pseudorange multipath mitigation technique, represents ways to take advantage of both code and carrier measurements [1–6].

CSC algorithm can alleviate the computational burden and reduce the communication bandwidth needed for the transmission of GPS observations. Therefore, different CSC algorithms have been developed for kinematic GPS applications. Amongst all the existing CSC algorithms, Hatch filter is the most well known and the simplest scheme. However, the Hatch filter is not optimal for the dual-frequency GPS data [4–6].

Based on the optimal parameter estimation theory, the improved Hatch filter and the optimal CSC algorithm for the dual-frequency GPS data are addressed in more detail. The optimal CSC algorithm, which is equivalent to the uncombined algorithm, can preserve the full information content of the raw dual-frequency GPS data. Theoretical development demonstrates that the optimal CSC algorithm has lower noise than the Hatch filter and its improved version.

2. Hatch filter

The dual-frequency GPS pseudorange and carrier phase measurements can be expressed in a concise matrix form [6–8]

$$\begin{pmatrix} R_1 \\ R_2 \\ L_1 \\ L_2 \end{pmatrix} = \begin{pmatrix} 1 & 1/f_1^2 & 0 & 0 \\ 1 & 1/f_2^2 & 0 & 0 \\ 1 & -1/f_1^2 & 1 & 0 \\ 1 & -1/f_2^2 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_\rho \\ A_1 \\ \lambda_1 N_1 \\ \lambda_2 N_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{P_1} \\ \varepsilon_{P_2} \\ \varepsilon_{L_1} \\ \varepsilon_{L_2} \end{pmatrix} \quad (1)$$

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with

$$A_1 = 40.28\,\text{TEC} \tag{2}$$

where f_i, λ_i, N_i are the frequency, wavelength and ambiguity parameter of $L_i\,(i = 1, 2)$, respectively; C_ρ is the non-dispersive delay, including geometric delay, tropospheric delay, clock biases and any other delay which affect all the observations identically; TEC (total electron content) is the integrated electron density along the signal path [8–10].

By applying the least-squares principle to Eq. (1), we can obtain

$$\lambda_1 \widehat{N}_1 = L_1 - (h_1 - h_2)R_1 - 2h_2 R_2 \tag{3a}$$

$$\lambda_2 \widehat{N}_2 = L_2 - 2h_1 R_1 + (h_1 - h_2)R_2 \tag{3b}$$

where $h_1 = \frac{f_1^2}{f_1^2 - f_2^2}$, and $h_2 = -\frac{f_2^2}{f_1^2 - f_2^2}$. These two linear combinations are constants in time, and so they are referred to as the time-invariant combinations. Obviously, the time-invariant combination is ionosphere-free and geometry-free; moreover, any combination of two time-invariant combinations is still time-invariant. Therefore, we can obtain a new time-invariant combination as follows:

$$\lambda_{\text{IF}} \widehat{N}_{\text{IF}} = L_{\text{IF}} - R_{\text{IF}} \tag{4}$$

with

$$\lambda_{\text{IF}} = \frac{c}{f_1^2 - f_2^2}, \quad N_{\text{IF}} = f_1 N_1 - f_2 N_2, \quad L_{\text{IF}} = h_1 L_1 + h_2 L_2,$$
$$R_{\text{IF}} = h_1 R_1 + h_2 R_2$$

where c is the speed of light in a vacuum.

The time averaging algorithm of the above time-invariant combination reads

$$\lambda_{\text{IF}} \overline{N}^n_{\text{IF}} = \frac{1}{n} \sum_{j=1}^{n} (L^j_{\text{IF}} - R^j_{\text{IF}}) = \frac{n-1}{n} \cdot \lambda_{\text{IF}} \overline{N}^{n-1}_{\text{IF}} + \frac{1}{n}(L^n_{\text{IF}} - R^n_{\text{IF}}) \tag{5}$$

where $n\,(\geqslant 2)$ is the width of the smoothing window. It follows immediately that

$$\overline{R}^n_{\text{IF}} = L^n_{\text{IF}} - \lambda_{\text{IF}} \overline{N}^n_{\text{IF}} = \frac{n-1}{n}(\overline{R}^{n-1}_{\text{IF}} + L^n_{\text{IF}} - L^{n-1}_{\text{IF}}) + \frac{R^n_{\text{IF}}}{n}. \tag{6}$$

This is the famous Hatch filter [1–6].

The Hatch filter formula (6) can be rewritten as

$$\overline{R}^n_{\text{IF}} = \frac{1}{n} \sum_{j=1}^{n} R^j_{\text{IF}} + \frac{1}{n} \sum_{j=1}^{n}(L^n_{\text{IF}} - L^j_{\text{IF}}) \tag{7}$$

for $\overline{R}^1_{\text{IF}} = R^1_{\text{IF}}$. Obviously, the low-frequency component of code measurements is extracted using the time averaging algorithm, while the time difference technique is employed to retrieve the high-frequency component of carrier measurements.

The Hatch filter is the most popular and simplest scheme. However, the Hatch filter is sub-optimal for the dual-frequency GPS data [6] in the sense of minimal variance.

3. Improved hatch filter

The improved Hatch filter should have lower noise than that of the Hatch filter. Moreover, it is an unbiased estimate of the satellite-receiver range of current epoch. Therefore, a class of carrier smoothing strategies can be constructed as follows:

$$\overline{R}^n_{\text{IF}}(\theta) = (1 - \theta)R^n_{\text{IF}} + \theta(L^n_{\text{IF}} - \lambda_{\text{IF}} \overline{N}^n_{\text{IF}}) \tag{8}$$

where θ is an arbitrary real value.

For simplicity, we assume that the same type of measurements have the same precision. Thus, the variance of the output in the filter (8) is

$$\text{cov}[\overline{R}^n_{\text{IF}}(\theta)] = \frac{n-1}{n} \sigma^2_{R_{\text{IF}}} \left[(1 + \gamma)\theta^2 - 2\theta + \frac{n}{n-1}\right], \tag{9}$$

where γ is the phase-code variance ratio, and $\sigma^2_{R_{\text{IF}}}$ is the variance of the combining measurement R_{IF}. It is easy to verify that, if and only if $\theta^* = (1 + \gamma)^{-1}$, the corresponding filter

$$\overline{R}^n_{\text{IF}}(\theta^*) = \frac{1}{1 + \gamma}(\gamma R^n_{\text{IF}} + L^n_{\text{IF}} - \lambda_{\text{IF}} \overline{N}^n_{\text{IF}}) \tag{10}$$

has minimal variance.

The expressions

$$\text{cov}\left[\overline{R}^n_{\text{IF}}(\theta^*)\right] = \frac{n-1}{n} \sigma^2_{R_{\text{IF}}} \left(\frac{\gamma}{1 + \gamma} + \frac{1}{n-1}\right) \tag{11}$$

and

$$\text{cov}(\overline{R}^n_{\text{IF}}) = \frac{n-1}{n} \sigma^2_{R_{\text{IF}}} \left(\gamma + \frac{1}{n-1}\right) \tag{12}$$

state that the improved version overperforms the Hatch filter. The precision gain indicator $\text{cov}(\overline{R}^n_{\text{IF}})/\text{cov}[\overline{R}^n_{\text{IF}}(\theta^*)]$ can be used to evaluate the performance of the improved filter (Fig. 1).
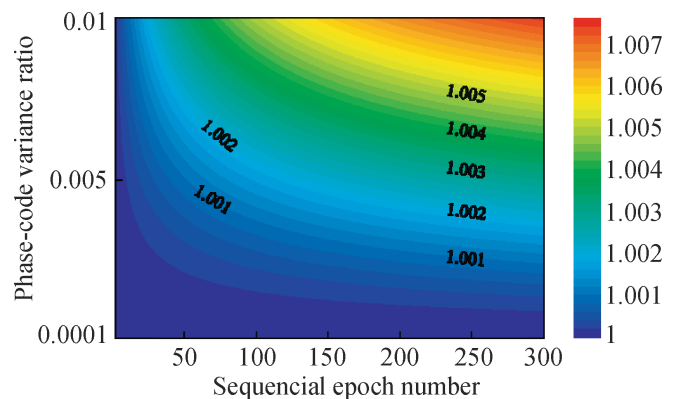

Fig. 1. The precision gain of the improved Hatch filter.

However, the improved filter is slightly superior to the popular Hatch filter. In fact, the limitation of the precision gain of the improved Hatch filter reads

$$\lim_{n\to\infty} \frac{\mathrm{cov}(\overline{R}_{\mathrm{IF}}^n)}{\mathrm{cov}[\overline{R}_{\mathrm{IF}}^n(\theta^*)]} = 1 + \gamma. \tag{13}$$

4. Optimal CSC algorithm for dual-frequency GPS data

In Hatch filter and its improved version, only the code–code, carrier–carrier ionosphere-free combinations are used, namely, some useful information is lost. Consequently, both of them are not globally optimal.

For dual-frequency GPS data, the previous studies [11] have shown that R_{IF}^n is the best linear unbiased estimate of the non-dispersive delay of current epoch. Therefore, the global optimal filter can be constructed as

$$\overline{R}_{\mathrm{IF}}^n(\theta_1, \theta_2) = R_{\mathrm{IF}}^n + \theta_1\lambda_1(\widehat{N}_1^n - \overline{N}_1^n) + \theta_2\lambda_2(\widehat{N}_2^n - \overline{N}_2^n) \tag{14}$$

where θ_1 and θ_2 are two arbitrary real values.

Substituting the recursive formulation

$$\lambda_i\overline{N}_i^n = \frac{1}{n}\lambda_i\widehat{N}_i^n + \frac{n-1}{n}\lambda_i\overline{N}_i^{n-1}, \quad i = 1, 2$$

into Eq. (14), we can obtain

$$\overline{R}_{\mathrm{IF}}^n(\theta_1, \theta_2) = R_{\mathrm{IF}}^n + \frac{n-1}{n}(\theta_1\lambda_1\widehat{N}_1^n + \theta_2\lambda_2\widehat{N}_2^n) - \frac{n-1}{n}$$
$$\times (\theta_1\lambda_1\overline{N}_1^{n-1} + \theta_2\lambda_2\overline{N}_2^{n-1}). \tag{15}$$

Denote

$$y_1 = -(h_1 - h_2)\theta_1 - 2h_1\theta_2, \quad y_2 = -2h_2\theta_1 + (h_1 - h_2)\theta_2.$$

Eq. (14) can be rewritten as

$$\overline{R}_{\mathrm{IF}}^n(\theta_1, \theta_2) = \frac{n-1}{n}(\theta_1 L_1^n + \theta_2 L_2^n) + \left(h_1 + \frac{n-1}{n}y_1\right)R_1^n$$
$$+ \left(h_2 + \frac{n-1}{n}y_2\right)R_2^n - \frac{1}{n}\sum_{j=1}^{n-1}(\theta_1 L_1^j + \theta_2 L_2^j$$
$$+ y_1 R_1^j + y_2 R_2^j).$$

When time-correlation [12,13] is absent, we have

$$\mathrm{cov}[\overline{R}_{\mathrm{IF}}^n(\theta_1, \theta_2)] = \frac{n-1}{n}\sigma_P^2$$
$$\times \left((\theta_1^2 + \theta_2^2)\gamma + (h_1 + y_1)^2 + (h_2 + y_2)^2 + \frac{h_1^2 + h_2^2}{n-1}\right) \tag{16}$$

from which we get the following condition equations

$$\begin{pmatrix} \gamma + (h_1 - h_2)^2 + 4h_2^2 & 2(h_1 - h_2)^2 \\ 2(h_1 - h_2)^2 & \gamma + (h_1 - h_2)^2 + 4h_1^2 \end{pmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$
$$= \begin{pmatrix} 2h_2^2 + h_1(h_1 - h_2) \\ 2h_1^2 - h_2(h_1 - h_2) \end{pmatrix}. \tag{17}$$

Thus, we have

$$\begin{cases} \theta_1^* = \frac{(h_1 - h_2)^2\gamma + h_2\gamma + h_1}{(1+\gamma)^2 + 4(h_1 - h_2)^2\gamma} \\ \theta_2^* = \frac{(h_1 - h_2)^2\gamma + h_1\gamma + h_2}{(1+\gamma)^2 + 4(h_1 - h_2)^2\gamma} \end{cases}. \tag{18}$$

Amongst all the linear combinations for the dual-frequency GPS data, $\overline{R}_{\mathrm{IF}}^n(\theta_1^*, \theta_2^*)$ has minimal variance, thus preserving the full information content [6]. Consequently, the best linear combination is equivalent to the uncombined dual-frequency GPS data.

Since

$$\mathrm{cov}[\overline{R}_{\mathrm{IF}}^n(\theta_1^*, \theta_2^*)] = \frac{n-1}{n}\sigma_{R_{\mathrm{IF}}}^2\left(\frac{(1+\gamma)\gamma}{(1+\gamma)^2 + 4(h_1 - h_2)^2\gamma} + \frac{1}{n-1}\right). \tag{19}$$

we can obtain

$$\lim_{n\to\infty} \frac{\mathrm{cov}(\overline{R}_{\mathrm{IF}}^n)}{\mathrm{cov}[\overline{R}_{\mathrm{IF}}^n(\theta_1^*, \theta_2^*)]} \approx 1 + 67.96\gamma. \tag{20}$$

The precision gain of optimal CSC algorithm is determined by the width of the smoothing window and the phase-code variance ratio (Fig. 2).

Systematic errors can be handled by the compensation of functional model or the refinement of stochastic model [7,14–16]. When the ambiguity parameters are taken as constant systematic errors, in the limitation form, the same results can be obtained through refinement of stochastic model [6].

5. Conclusion

(1) Hatch filter is the simplest scheme. The optimal CSC algorithm can provide more precise positioning results; however, the phase-code variance ratio should be determined properly.
(2) When cycle slips occur, the filter has to be restarted. Cycle slips can be detected using the integrated Doppler, which is more reasonable and effective than the differential phase [2,8].
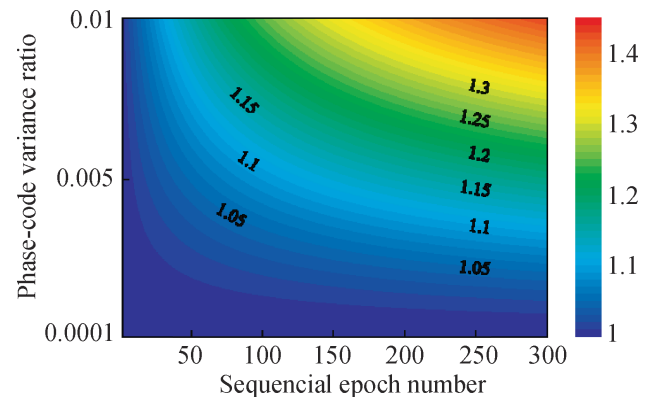

Fig. 2. The precision gain of the optimal filter.

(3) According to Eqs. (11), (12), and (19), all the smoothed measurements can be strictly weighted. Thus, the "abrupt sting" phenomenon can be eliminated [17].

Acknowledgements

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